

**OPTIMAL ALLOCATION OF AIR STRIKES FOR  
INTERDICTION OF A TRANSPORTATION NETWORK**

**THOMAS MORTON MUSTIN**

7-15 507307  
507307  
507307  
DUDLEY KNOX LIBRARY  
NAVAL POSTGRADUATE SCHOOL  
MONTEREY CA 93943-5101









OPTIMAL ALLOCATION OF AIR STRIKES  
FOR INTERDICTION OF A TRANSPORTATION NETWORK

by

Thomas Morton Mustin  
Lieutenant, United States Navy  
B.S., Naval Academy, 1962

~~RELEASABLE TO NDS INTERNATIONAL AFFILIATES~~

~~(Ref.: Memo, 1702 NC4(0142) dtd 21 Mar 68)~~

~~TOP SECRET~~

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL  
June 1967

M17865  
C.1

## ABSTRACT

Under certain conditions, the re-supply capability of a combatant force may be limited by the characteristics of the transportation network over which supplies must flow. Air strikes by an opposing force may be used to reduce the capacity of that network; the effects of such strikes vary for differing missions and targets. With only a limited number of sorties available, the strike planner must decide which targets to hit, and with how many sorties. A computational procedure is developed for determining the optimum strike plan for minimizing network flow capacity.



TABLE OF CONTENTS

Section	Page
I. Introduction	9
General	9
The Strike Planning Problem	12
II. Objective and Scope	14
III. The Model	15
IV. Solution Procedure	20
Preview	20
The $r$ th Shortest Route	21
Shortest Route Algorithm	22
The $r$ th Shortest Route Algorithm	23
The SIRL Algorithm	25
Example	27
V. Discussion	32
Finiteness	32
Sensitivity Analysis	32
Modification for $L_{ij} = 0$	34
Assumptions	36
Applications	38
Recommendations for Further Study	39
VI. Summary	40
Bibliography	42



## LIST OF ILLUSTRATIONS

Figure	Page
1. An Example Network	28
2. The Topological Dual	29
3. Route Length versus Number of Sorties	33
4. Arc Capacity versus Number of Sorties	37



# LIST OF SYMBOLS

$(i,j)$	-an arc connecting nodes $i$ and $j$
$U_{ij}$	-the maximum capacity of arc $(i,j)$ in tons per day
$L_{ij}$	-the minimum capacity of arc $(i,j)$ in tons per day
$m_{ij}$	-the actual capacity of arc $(i,j)$ in tons per day
$C_{ij}$	-the number of sorties required to reduce the capacity of arc $(i,j)$ by one unit
$K$	-the total number of sorties available for a given day
$x_{ij}$	-the actual flow existing in arc $(i,j)$
$Q$	-the total flow passing through a network per day
$S_r$	-the length of route $r$ , for any $m_{ij}$
$S_r^*$	-the length of route $r$ when all $m_{ij} = L_{ij}$
$S'_r$	-the minimum achievable value of $S_r$
$C_r$	-the number of sorties required to reduce the length of route $r$ to $S_r$
$C_r^*$	-the number of sorties required to reduce the length of route $r$ to $S_r^*$
$C'_r$	-the number of sorties required to reduce the length of route $r$ to $S'_r$
$R_r$	-the $r$ th shortest route from source to sink, with all $m_{ij}$ set = $L_{ij}$
$A_{rz}$	-the arc in route $r$ with the $z$ th highest value of $C_{ij}$



## I. INTRODUCTION

General: In a military campaign, the ability of one or both of the forces to carry on may depend directly on their ability to transport materiel some distance overland. This is particularly true in the case of armed aggression by one nation upon a contiguous or near-contiguous nation, and is accentuated when the aggressor is denied the use of sea communications, either by geographical considerations or by overwhelming sea supremacy on the part of the opponent.

The amount of materiel required to support a military force in the field may vary from as much as sixty pounds per man per day, as for the U.S. forces in Korea, to as low as perhaps two or three pounds per man per day for a hit-and-run guerilla force<sup>(2)</sup>. In any case, there is some minimum rate of supply required to sustain a combatant force; if the rate drops below this level the force must either reduce its size or curtail its activities. Accordingly, it is to the advantage of the opponent of such a force to reduce the resupply capability as much as possible.

An aggressor's ability to supply himself may be limited by one of three factors: the number of vehicles available for transporting goods from supply bases to the fighting force; the amount of goods on hand and available for shipment from supply bases to the field; or the nature of the transportation system between supply areas and the front. In <sup>extreme</sup> ease case, air strikes can be used to reduce the aggressor's re-supply capability.

Durbin<sup>(3)</sup> has outlined a procedure for determining maximum cargo flow as a function of available vehicles, and for sequentially selecting and destroying the most vital link in the transportation network until a predetermined number of links has been destroyed or until flow has been stopped. Wollmer<sup>(8,9)</sup> has developed a method for determining the n most vital links in a network, both when flow through the network is limited by the number of vehicles, and when it is limited by the network configuration itself; in both cases he assumed that the capacity of a given link can be reduced to zero. In none of these studies was the cost of interdiction considered.

This paper will address only the situation where flow through the transportation system is limited by the capacities of the various road and rail segments comprising the system, and will take into consideration the number of air strikes required to effect reduction in the capacity of each arc and the limitations on the total number of air strikes available. The time period considered will be one 24-hour day. Such a scenario is considered particularly apposite to the current war in Viet Nam, where the transportation system involved in the resupply of VC/VNA forces is not overly sophisticated, and political considerations have placed restrictions on the number of interdiction sorties available each day. Furthermore, since the same political considerations have dictated allowing the North Vietnamese to import both supplies and supply vehicles with impunity, it seems reasonable to assume that the supply capability



of the VC/VNA forces is limited by the transportation network used to transport goods into South Viet Nam.

Any particular stretch of road or rail has some upper limit on the number of tons per day of goods which it can support. In the case of roads, this upper limit is dependent upon surface type, surface conditions, pavement width, and terrain. Holliday<sup>(7)</sup> has prepared a detailed and comprehensive method for estimating such capacities. The capacity of a railroad, when not bounded by the number of available locomotives and rolling stock, may be limited by the characteristics of a particular bridge or trestle, by the type of roadbed, or by the service facilities available throughout the network.

The use of air strikes against a given stretch of road or rail reduces its capacity to some extent, whether by cratering a road, destroying railroad track, destroying fixed targets such as bridges and fuel or supply depots, or simply (as in the case of armed reconnaissance flights) by deterring the enemy from using that segment of road.

In general, the greater the number of sorties used against a section of road, the greater the reduction in its transportation capacity. Eventually, however, the principle of diminishing returns takes effect to the extent that, for practical purposes, additional sorties yield no further reduction. Experience<sup>(3,4,6)</sup> has indicated that this lower limit, measured in tons per day, is always greater than zero, as there are myriad methods available for use in countering an interdiction campaign, including, for example, the

construction of by-passes, shuttling traffic between road or rail cuts, camouflage, the use of coolie labor for repairs, anti-aircraft flak traps, elaborate warning systems, extensive pre-deployment of repair equipment, and deception-- such as removing a span from a bridge by day, making it appear unserviceable, and replacing it after nightfall<sup>(3)</sup>.

The most dramatic reduction in capacity as a result of air strikes occurs in railroads, owing to the vulnerability of track to destruction from the air. But even in this area, Korean experience showed that coolies, working at night, can repair a rail cut in eight hours.<sup>(6)</sup> Thus, even in the area of maximum returns per interdiction sortie, the force being interdicted still retains the ability to move some amount of goods over a given segment of its transportation system within 24 hours. This non-zero lower limit on segment capacity will later be seen to be important.

The Strike Planning Problem: Consider the problem facing the strike planner in determining what targets to hit on a given day. Assuming that his objective is interdiction of the enemy's supply lines, and that the enemy's supply capability is network-limited, he wishes to reduce the capacity of the network as much as possible. He knows the capacity of each segment of the transportation system in tons per day, and thus what the total network capacity is; furthermore, he knows, for each segment, the minimum value to which he can reduce the capacity, and the number of sorties required to accomplish this reduction. If there were no

limit on the number of sorties available to him, he could simply direct that each segment receive the number of sorties required to reduce its capacity to a minimum, and thus he would be assured that the network capacity had been reduced to the absolute minimum possible.

If, however, the number of sorties available to him is limited, he is faced with a problem of allocating scarce resources: how many sorties should be flown against each segment in the next 24 hours in order to reduce as much as possible the enemy's ability to transport goods during that period?

## II. OBJECTIVE AND SCOPE

The object of this study is to find a solution procedure for the strike planning problem. An algorithm will be developed for solving the problem under the assumptions that upper and lower limits on road capacities, as well as the amount of reduction per sortie, are known deterministically, and the reduction in capacity per sortie is linear between the upper and lower limits of capacity (although this restriction is by no means necessary, as will be demonstrated in Section V). No attempt will be made to take into account the vulnerability of the attacking aircraft, or the varying amounts of time required to restore flow capacity over road segments which have been attacked.

The solution procedure to be developed on the following pages uses as inputs the maximum capacity of each road segment, the minimum value to which that capacity can be reduced, the number of sorties required to effect such reduction, and the total number of sorties available for use. As outputs, it provides the set of road segments which should be attacked, the number of sorties to be used against each of these segments, and the total flow capacity of the network after attack.



### III. THE MODEL

The transportation system may be represented by a network of numbered nodes and associated arcs. Each arc  $(i,j)$  represents a section of road or rail, and is capable of passing a known quantity,  $U_{ij}$ , of supplies per day; such capability is assumed to be two-directional. A node may be a town, an intersection, or any place where it is useful to distinguish between road capacities on either side of the node.

The network is assumed to have a single source, node 1, and a single sink, node  $n$ . If, in fact, there is more than one source (or sink), an artificial node may be constructed, with an artificial arc connecting it with each actual source (or sink). The capacity of each of these artificial arcs can be any value which is greater than or equal to the sum of the capacities of the arcs leading out of the actual source node or into the actual sink node.

Each arc is represented by  $(i,j)$ , where the two integers,  $i$  and  $j$ , correspond to the numbers of the nodes which the arc connects. As flow capacity is two-directional, it is of no consequence which number is written first. However, when the context is that of an actual flow occurring in a particular direction, the designation  $(i,j)$  indicates that flow moves from  $i$  to  $j$ .

The capacity of arc  $(i,j)$  at any time is designated by  $m_{ij}$ . The value of  $m_{ij}$  is equal to  $U_{ij}$  if no air strikes are launched against arc  $(i,j)$ . The use of strikes against the arc may reduce  $m_{ij}$  down to a value of  $L_{ij}$  (but no lower),

or to any intermediate value. This reduction is assumed for the time being to be a linear function of the number of sorties. The amount of reduction per sortie may vary from arc to arc. The number of sorties required to reduce the capacity of arc  $(i,j)$  by one unit is  $C_{ij}$ ; the total number of sorties available on a particular day is  $K$ . Thus,

$$L_{ij} \leq m_{ij} \leq U_{ij} \text{ and}$$

$$\sum_{\text{all } (i,j)} (U_{ij} - m_{ij}) C_{ij} \leq K.$$

The actual flow passing over arc  $(i,j)$  is denoted by  $x_{ij}$ . The total flow passing through the network is  $Q$ . The problem of the force which is shipping supplies and materiel through the network is a standard maximum flow problem<sup>(5)</sup>:

$$\text{Maximize } Q = \sum_j x_{1j} = \sum_i x_{in} ,$$

$$\text{subject to } \sum_j x_{ij} - \sum_k x_{ki} = 0, \quad i = 2, 3, \dots, n-1$$

$$\text{and } 0 \leq x_{ij} \leq m_{ij} \leq U_{ij}, \quad \begin{matrix} i = 1, 2, \dots, n \\ j = 1, 2, \dots, n, i \neq j, \end{matrix}$$

where, for arcs  $(i,j)$  which do not exist, it is understood that  $U_{ij} = 0$ . The first set of constraints is a consequence of the principle of flow conservation at the nodes; i.e., the total flow out of any node must equal the total flow into that node.

The problem facing the interdicting force is not so easy to state: it is only indirectly concerned with minimizing the actual flow in the network. Under the assumption that flow is network-limited (and therefore not limited by the number of vehicles or the quantity of supplies at the

source), it is not necessarily true that the interdicator would wish to strike only at those arcs over which the existing flow is traveling. To do so would, of course, destroy some quantity of goods and vehicles on the road, but they are replaceable. Additionally, although the capacity of these arcs would be reduced, this reduction may or may not reduce the overall network capacity. The force moving the goods need only re-route his supply vehicles, and he has suffered no loss in his ability to transport goods. In short, the interdiction effort is aimed at minimizing flow capacity (thus, ultimately, the flow itself), rather than immediately minimizing flow.

In order to state the problem facing the interdicator, it is useful to introduce the notion of a cut set. A cut set, for a network of undirected arcs, is defined to be a set of arcs which, if removed from the network, separate the source from the sink. The value of a cut set is defined as the sum of the capacities of the arcs comprising the cut set. The minimum cut set of a network is that cut set whose value is the smallest.

The Max-flow Min-cut Theorem<sup>(5)</sup> states that the maximum possible flow through a network is equal to the value of the minimum cut set in the network. Thus the interdicting force seeks to use its sorties so as to minimize the minimum cut

set of the interdicted network, or

$$\begin{aligned}
 &\text{Minimize} && \sum_{(i,j) \in \text{cut set}} m_{ij}, \\
 &(\text{over all cut sets}) \\
 &\text{subject to} && \sum_{\substack{\text{all} \\ i,j}} (U_{ij} - m_{ij}) C_{ij} \leq K, \quad \begin{matrix} i = 1, 2, \dots, n \\ j = 1, 2, \dots, n, i \neq j \end{matrix} \\
 &&& \text{and } L_{ij} \leq m_{ij} \leq U_{ij} .
 \end{aligned}$$

A helpful device in the solution of this problem is the notion of the topological dual of a network. A topological dual may be constructed as follows<sup>(9)</sup>: to the original (primal) network add an artificial arc connecting source to sink. Place a node in each mesh of the primal; let the source of the topological dual be the node in the mesh above the artificial arc and the sink the node below it. For each arc in the primal, except the artificial arc, construct an arc that intersects it and joins the nodes in the meshes on either side of it. The lengths of the arcs in the topological dual correspond to the capacities of the arcs which they intersect in the primal. The values of  $U_{ij}$ ,  $L_{ij}$ , and  $C_{ij}$  are identical to those of the primal arcs, and they now represent the upper and lower limits of arc length, and the cost of reducing arc length by one unit.

Each route through the topological dual corresponds to some cut set of the primal. Thus the problem of minimizing the minimum cut set (after interdiction) becomes one of minimizing the post-interdiction shortest route through the topological dual.



It should be noted that the concept of the topological dual applies only for planar networks; i.e., those which can be represented on a plane such that no two arcs intersect except at a node. Land transportation networks are generally planar networks, particularly in the case of rural systems such as that in Viet Nam.

#### IV. SOLUTION PROCEDURE

Preview: The algorithm presented below solves the strike planning problem in the following manner: the topological dual is first constructed, and all arc lengths are considered reduced to their minimum values. The shortest route through the topological dual and the number of sorties required to reduce it to this length are then determined. If the sorties required are not more than those available, the problem is solved; the primal network capacity can be reduced to the value of this length and no lower. If the sorties required to effect this reduction are more than are available, we systematically reduce the number of sorties flown, concurrently increasing route length (this procedure can be likened to "un-flying" the requisite number of sorties). We begin the un-flying procedure with that arc which requires the highest number of sorties per unit reduction, then proceed to the arc requiring the second-highest number, etc., until the total sorties flown equals the number available. The minimum value to which this particular route can be reduced has now been determined.

The second shortest route is then determined, and the above procedure repeated for it, and then the third shortest, etc. The algorithm terminates when either of two conditions arises:

- a) A route is found whose minimum length can be attained with the available sorties. All remaining routes (with the exception of ties) have minimum values greater than the one just

examined, and thus no further routes need be examined (again, except for ties).

- b) A route is found such that the final length of some previously considered route (after the necessary increases in route length have been made by un-flying the required number of sorties) is less than the minimum length of this route. No further routes need be examined, as all those remaining have minimum lengths greater than a route which has already been found achievable with the available sorties.

If, at any time during the process of increasing route length by un-flying sorties, the length of a route exceeds the final route length for some previously considered route, the particular route under examination need be considered no further (although, of course, subsequent routes must still be examined until one of the conditions above has been met).

The  $r$ th Shortest Route: An integral part of the algorithm is the determination of the  $r$ th shortest route through the topological dual. A procedure for finding the  $r$ th shortest route is given below.

It is first necessary to determine the shortest route from the source to each node. The general problem of

route minimization can be stated as (5)

$$\text{Minimize } \sum_{\substack{\text{(all} \\ \text{arcs)}}} m_{ij} x_{ij} ,$$

$$\text{subject to } EX = \begin{vmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ -1 \end{vmatrix} ,$$

$$\text{and } x_{ij} \geq 0 \text{ for all arcs,}$$

where E represents the node-arc incidence matrix of the network, and X is the vector of  $x_{ij}$ 's. It is useful to state the dual of this problem, which is to

$$\text{Maximize } V_1 - V_n$$

$$\text{subject to } E^T V \leq m^T ,$$

where V is the column vector of the dual variables  $V_i$  (which are unrestricted in sign), and m is the row vector of  $m_{ij}$ 's. The following algorithm is based on this dual problem.

Shortest Route Algorithm: To find the shortest route to any node,

- 1) Start with any feasible dual (for example,  $V_i = 0$  for all  $i$  if  $m_{ij} \geq 0$  for all  $(i,j)$  ).
- 2) For node  $j$ , scan all arcs  $(i,j)$  and set

$$V_j = \min_i (m_{ij} + V_i) .$$

Repeat this step until all nodes have been inspected with no change in  $V_j$  values.

3) A tree has been found consisting of all arcs having  $V_j - V_i = m_{ij}$ . The trunk of this tree connects the source with the sink and is the minimum route.

The  $r$ th Shortest Route Algorithm: The following algorithm for determining the  $r$ th shortest route is suggested by Bellman and Dreyfus.<sup>(1)</sup>

Let  $V_j^{(r)}$  denote the  $r$ th shortest route from the source to node  $j$ .  $V_1^{(r)} \equiv 0$ . Further, let

$\min_1 (\cdot)$  denote the absolute minimum of  $(\cdot)$ ,  
 $\min_2 (\cdot)$  denote the second smallest value of  $(\cdot)$ ,  
 $\vdots$   
 $\min_i (\cdot)$  denote the  $i$ th smallest value of  $(\cdot)$ .

The shortest route from the source to node  $j$  is then  $V_j = \min_i (V_i + m_{ij})$ .

The second shortest route is

$$V_j^{(2)} = \min \begin{bmatrix} \min_1 (V_i^{(2)} + m_{ij}) \\ \min_2 (V_i^{(1)} + m_{ij}) \end{bmatrix}$$

In general, for the  $r$ th shortest route we may write

$$V_j^{(r)} = \min \begin{bmatrix} \min_1 (V_i^{(r)} + m_{ij}) \\ \min_2 (V_i^{(r-1)} + m_{ij}) \\ \vdots \\ \min_{r-1} (V_i^{(2)} + m_{ij}) \\ \min_r (V_i^{(1)} + m_{ij}) \end{bmatrix}$$

To find the  $r$ th shortest route from the source to node  $j$  we begin by finding the shortest route using the algorithm given above. Next we find the second shortest route by modifying the second step of the shortest route algorithm (replace

$$V_j = \min_i (m_{ij} + V_i) \text{ by } V_j^{(2)} = \min \left[ \begin{array}{l} \min_1 (V_i^{(2)} + m_{ij}) \\ \min_2 (V_i^{(1)} + m_{ij}) \end{array} \right] ).$$

Then we find the third shortest route, and so on. The algorithm terminates after we obtain a value of  $V_n^{(r)}$  and the path from source to sink which generates this value.



The SIRL Algorithm: In the presentation of the SIRL (Selectively Increasing Route Length) Algorithm, a feasible route length is defined as one which can be achieved by using no more than the available sorties. To facilitate the presentation, the following variables are defined:

$K$  = the total number of sorties available for use,

$S_r$  = the length of route  $r$ , for any  $m_{ij}$ ,

$S_r^*$  = the length of route  $r$  when all  $m_{ij} = L_{ij}$ ,

$S'_r$  = the minimum feasible value of  $S_r$ ,

$C_r$  = the number of sorties required to reduce the length of route  $r$  to  $S_r$ ,

$C_r^* = \sum_{(i,j) \in R_r} (U_{ij} - L_{ij}) C_{ij}$ , the number of sorties required to reduce the length of  $R_r$  to  $S_r^*$ ,

$C'_r$  = the number of sorties required to reduce  $S_r$  to  $S'_r$  (if  $S'_r = S_r^*$ , then  $C'_r = C_r^* \leq K$ ).

Otherwise,  $C'_r = K$ ).

$R_r$  denotes the  $r$ th shortest route from source to sink, with all  $m_{ij} = L_{ij}$ , and  $A_{rz}$  denotes the arc with the  $z$ th highest value of  $C_{ij}$  in route  $r$ . The expression  $(U_{ij} - L_{ij})_{rz}$  is equivalent to  $(U_{ij} - L_{ij})$  for arc  $A_{rz}$ .

The algorithm proceeds as follows:

1. Construct the topological dual of the network.

Set  $r = 0$ ; set all  $m_{ij} = L_{ij}$ .

2. Set  $r = r + 1$ . Determine  $R_r$  and its length  $S_r^*$ .  
If  $S_r^* > \text{some previous } S_i'$ , go to step 7.
3. Compute  $C_r^*$ . If  $C_r^* > K$ , go to step 4. If  $C_r^* \leq K$ , determine  $R_i$ ,  $S_i^*$ , and  $C_i^*$  for  $i = r + 1$ ,  $r + 2$ , ..., until some  $S_i^* > S_r^*$  (to insure inclusion of all ties). Ignore any route for which  $C_i^* > K$ . Go to step 7.
4.  $Y = (C_r^* - K)$  sorties must be un-flown. Rank the arcs in  $R_r$  in descending order of  $C_{ij}$ . Set  $Z = 0$ .
5. Set  $z = z + 1$ . For  $A_{rz}$ , increase arc length to  $\min(U_{ij}; L_{ij} + \frac{Y}{C_{ij}})$ .
  - a) If  $U_{ij} < L_{ij} + \frac{Y}{C_{ij}}$ , go to step 6.
  - b) If  $U_{ij} \geq L_{ij} + \frac{Y}{C_{ij}}$ ,  $S_r$  has been increased to  $S_r' = S_r + \frac{Y}{C_{ij}}$ .  $C_r' = C_r - Y = K$ . Return to step 2.
6. Route length has been increased such that  $S_r = S_r + (U_{ij} - L_{ij})_{rz}$ .
  - a) If  $S_r > \text{some previous } S_i'$ , disregard  $R_r$  from further consideration and return to step 2.
  - b) If  $S_r \leq \text{all previous } S_i'$ ,  $Y - (U_{ij} - L_{ij})_{rz} C_{ij}$  still remains to be unflown.  
Set  $Y = Y - (U_{ij} - L_{ij})_{rz} C_{ij}$  and return to step 5.



7. Compare all values of  $S'_i$  and select the route for which  $S'_i$  is a minimum. In case of ties, select the route for which  $C'_i$  is smallest. The value of  $S'_r$  represents the minimum achievable network capacity. The number of sorties required to effect reduction to this level is  $C'_r$ . The number of sorties to be flown against each arc of this route (and therefore against each corresponding arc in the primal cut set) is  $(U_{ij} - L_{ij}) C_{ij}$ , except for those arcs whose lengths were increased by step 5. For these arcs, the number of sorties to be flown is

$$C_{ij} \left[ U_{ij} - \min \left( U_{ij}; L_{ij} + \frac{Y}{C_{ij}} \right) \right],$$

where  $Y$  is the last value of  $Y$  obtained in step 6.

Example: Consider the transportation network of Figure 1.

The three numbers shown for each arc represent  $U_{ij}$ ,  $L_{ij}$ , and  $C_{ij}$ , respectively. Suppose, for this problem, that  $K = 80$  sorties.

The asterisks represent the nodes of the topological dual, and the dashed lines its arcs. Note the inclusion of the artificial source-to-sink arc  $(9,1)$  for use in constructing the topological dual; note also that the numbers on the nodes of the topological dual are unrelated to the node numbers of the primal.

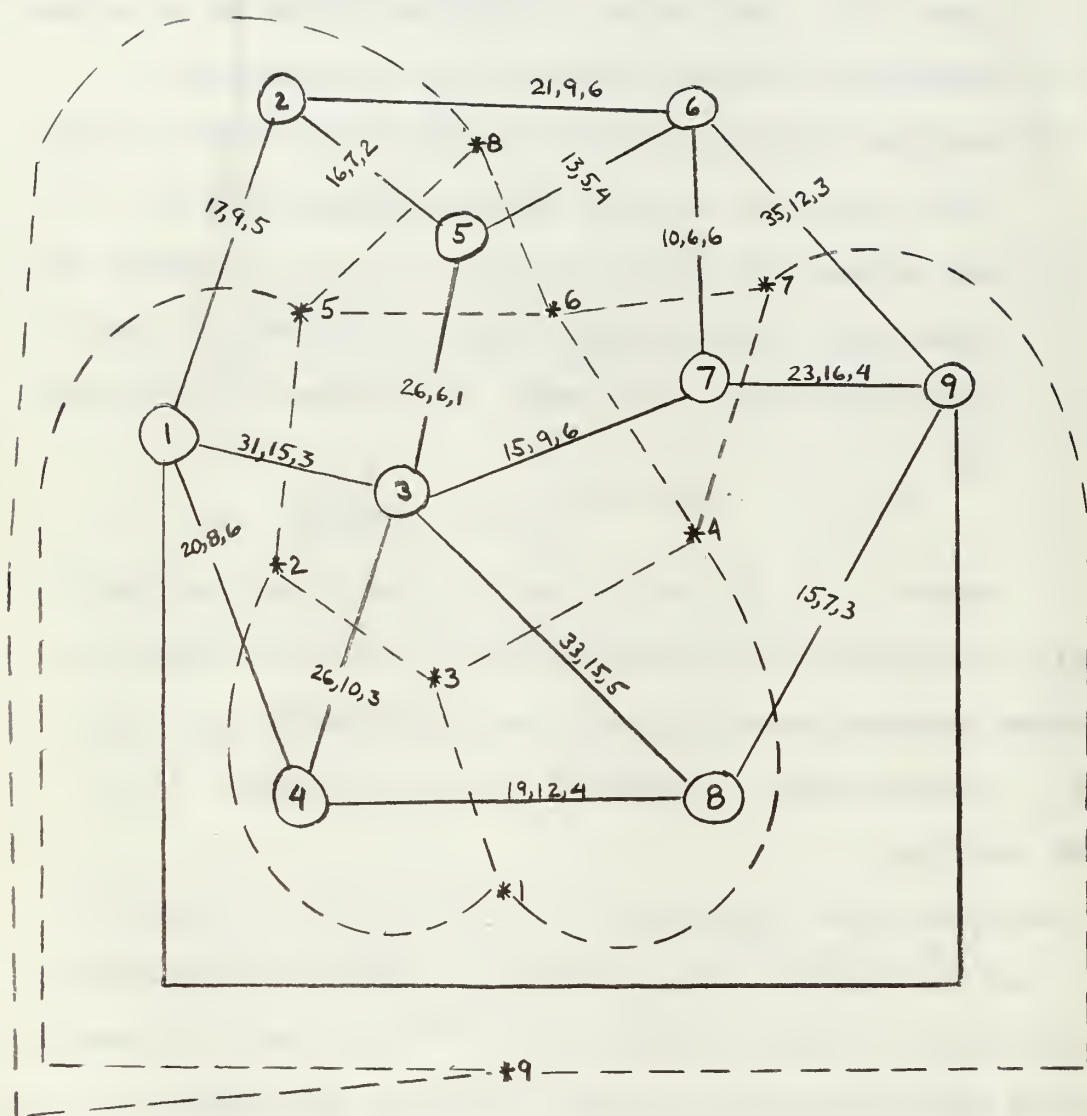


Figure 1. An Example Network

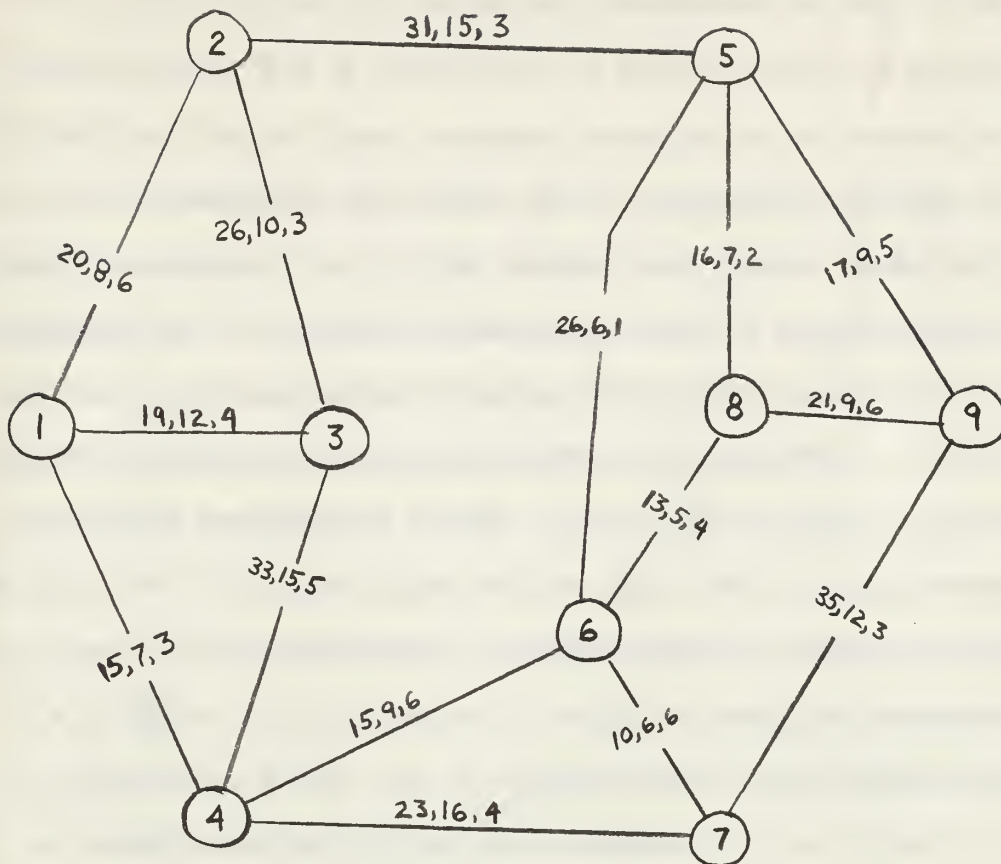


Figure 2. The Topological Dual

Figure 2 shows the topological dual, by itself, with the "transferred" arc information. The minimum-length route through this network before any interdiction has taken place (1,4,6,8,9), with a length of 64 units. This length is equivalent to the value of the maximum flow possible through the primal network.

Proceeding in accordance with the algorithm, we first reduce all arc lengths to their  $L_{ij}$  values. The shortest route,  $R_1$ , is then found to be (1,4,6,8,9); its length is

$S_1^* = 30$ . The corresponding value of  $C_1^*$  is  $(15 - 7) \times 3 + (15 - 9) \times 6 + (13 - 5) \times 4 + (21 - 9) \times 6 = 164$  sorties. As this value is 84 sorties greater than the 80 available, we must un-fly 84 sorties. We find that the two arcs (4,6) and (8,9) have identical values of  $C_{ij} = 6$ , which are greater than the  $C_{ij}$ 's of all other arcs in  $R_1$ . It is inconsequential which of these two arcs we select as  $A_{11}$ ; let us arbitrarily choose (8,9). We increase the length of this arc to  $\min(21; 9 + \frac{84}{6}) = 21$ . Route length has now been increased to  $S_1 = 30 + (21 - 9) = 42$ .  $84 - (21 - 9) \times 6 = 12$  sorties remain to be un-flown. Selecting (4,6) as  $A_{12}$ , we increase the length of (4,6) to  $\min(15; 9 + \frac{12}{6}) = 11$ . This increases the length of  $R_1$  to 44, while reducing  $C_1$  to its final (and feasible) value of  $C_1^* = 80$  sorties. Thus 44 is the minimum length to which this route can feasibly be reduced.

We next determine  $R_2$  to be (1,4,6,5,9), resulting in  $S_2^* = 31$  and  $C_2^* = 120$  sorties. Again, some sorties must be un-flown.  $A_{21}$  is seen to be (4,6) with  $C_{46} = 6$ . We increase this arc's length to  $U_{46} = 15$ , thereby increasing  $S_2$  to 37, and leaving 4 sorties to be un-flown.  $A_{22}$  is (5,9);  $C_{59} = 5$ . We achieve feasibility ( $S_2' = 80$ ) by increasing the length of (5,9) by 0.8 units, resulting in  $S_2' = 37.8$

$R_3$  is next determined; it is (1,2,5,9).  $S_3^* = 32$ , and  $C_3^* = 160$ . Increasing the length of  $A_{31}$  (arc (1,2)) to its maximum value of  $U_{12} = 20$ , we increase  $S_3$  to 44. Though our  $C_3$  is still at the infeasible value of 88, we need

proceed no further with this route, as it has already been increased in length to where it is equal to  $S_1'$  and greater than  $S_2'$

$R_4$  is found to be  $(1,4,6,7,9)$ , with  $S_4^* = 34$  and  $C_4^* = 153$  sorties. Selecting  $(6,7)$  as  $A_{41}$ , we increase  $S_4$  to a value of 38 while reducing  $C_4$  to 88. Again, while we still have not reached feasibility,  $S_4$  is greater than  $S_2'$ ; therefore we can proceed to the next route.

We next find  $R_5$  to be  $(1,4,7,9)$ . We compute  $C_5^* = 129$  and  $S_5^* = 35$ . Decreasing  $C_5$  to 93 sorties increases  $S_5$  to 42, which is greater than  $S_2'$ ; thus we go to the next route.

$R_6$ , which is  $(1,4,6,5,8,9)$ , has a minimum length of  $S_6^* = 38$ . As this and all subsequent  $S_i^*$  are greater than  $S_2'$ , we need examine no further routes.

Comparing the  $S_1'$ , we see that the minimum  $S_1'$  is  $S_2' = 37.8$ , which is therefore the minimum value to which we can reduce the capacity of the primal network. This reduction is obtained by flying 24 sorties against arc  $(8,9)$ , 20 sorties against arc  $(5,6)$ , and 36 sorties against arc  $(2,6)$  in the original network.



## V. DISCUSSION

Finiteness: The algorithm terminates in a finite number of steps, since in no case is it necessary to examine any route more than once, and the number of routes in a finite network is finite. The algorithm converges toward the point where it will terminate, since for each successive iteration the minimum value of the  $S_i^!$ 's thus far determined is less than or equal to its value at the end of the previous iteration, each successive  $S_i^*$  is greater than or equal to its predecessor, and the algorithm terminates when some  $S_i^*$  is greater than the minimum of the previous  $S_i^!$ 's.

Sensitivity Analysis: One convenient feature of this solution procedure is that the SIRL algorithm generates, for each route through the topological dual, simultaneous values of  $S_i$  and  $C_i$ , which can be used for a plot of sensitivity of minimum achievable route length versus available sorties. The value of  $K$  can be specified as low as desired in order to determine the optimal sortie program over a wide range of available sorties. Such a procedure may be very useful to the strike planner: it may reveal, for example, that some small increase in the number of available sorties would dictate an entirely different strike plan, and would allow significantly better reduction in network capacity.

Figure 3 is a plot of route length (hence, network capacity) versus sorties for routes  $R_1$  through  $R_3$  of the example problem. The value of  $K$  ranges from zero to the

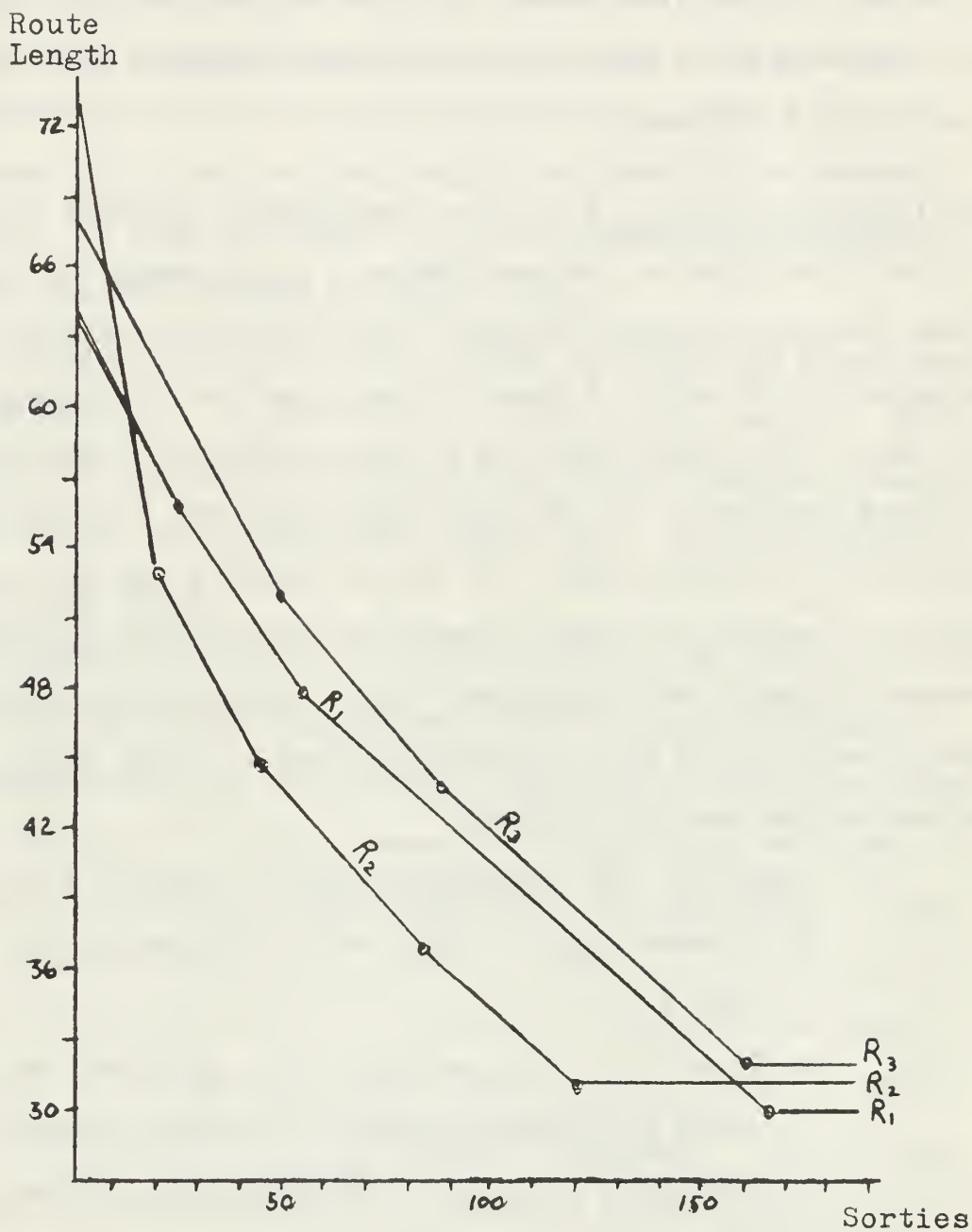


Figure 3. Route Length vs. Number of Sorties

point where no further reduction can be obtained on any of the routes considered. Note that, in this example,  $R_2$  is the route which would receive air strikes for  $12 < K < 160$ . Outside this range of  $K$  values,  $R_1$  can be reduced further than  $R_2$ . It is also worthy of note that we get virtually no improvement in our ability to reduce network capacity for  $K > 120$  sorties.

Modification for  $L_{ij} = 0$ : One immediately apparent weakness of the algorithm is the case where a large number of routes have identical minimum values, which could necessitate the examination of each of these routes, particularly when  $K$  is small. The occurrence of a large number of identical minimum lengths is highly improbable when the  $L_{ij}$  (or most of the  $L_{ij}$ ) are non-zero. If all or most of the  $L_{ij}$  are zero, of course, we have a number of zero-length minimum routes through the topological dual. Such an eventuality could be provided for by modifying step 1 of the SIRL algorithm to read as follows:

1. Construct the topological dual. Examine all  $L_{ij}$ 's.
  - a) If some  $L_{ij} > 0$ , then set  $r = 0$  and go to step 2.
  - b) If all  $L_{ij} = 0$ , set  $m_{ij} = U_{ij}$  and find the route of minimum length in the dual network. Calculate  $C^*$  for this minimum route where

$$C^* = \sum_{\substack{(i,j) \\ \text{min route}}} U_{ij} C_{ij} .$$



If  $C^* > K$ , set  $r = 0$ , set all  $m_{ij} = L_{ij}$ ,  
and go to step 2.

If  $C^* \leq K$ , terminate.

The effect of this modification is that we find the shortest uninterdicted route and see if we feasibly can reduce it to its minimum value, i.e., zero. If not, we then proceed to the standard algorithm and examine routes at random until one is found which can feasibly be reduced to zero. If none is found, of course, we must determine minimum feasible length of every route.

Suppose, in using step 1 (b) of the modified algorithm, we find that  $C^* > K$ , but only by a very small amount. Then it would be worthwhile examining the second shortest uninterdicted route, and perhaps the third, etc., until hopefully a  $C^* \leq K$  is found.

It should be noted that if, in step 1 (b), a  $C^* \leq K$  is found, the algorithm terminates with a feasible reduction of network capacity to zero, but some other subsequent value of  $C^*$ , which will remain unexamined, might very well be smaller than the one which terminated the algorithm. Thus under these conditions we obtain a feasible solution, but are not guaranteed that it is the one requiring the least sorties. A similar condition prevails if, failing to find a  $C^* \leq K$ , we return to the standard algorithm: as soon as we find any feasible means of reducing network capacity to zero, we terminate, even though some other combination of fewer sorties might also reduce it to zero.

Incorporating the modification outlined above, where necessary, would hopefully eliminate the necessity of examining every route except in cases of very small  $K$ .

Assumptions: The assumption of deterministically known values of  $U_{ij}$ ,  $L_{ij}$ , and  $C_{ij}$  is highly artificial, particularly for  $L_{ij}$  and  $C_{ij}$ . In reality, these quantities are random variables with unknown distributions.

The quantity  $U_{ij}$  can be determined most easily (using Holliday's method), but  $L_{ij}$  and  $C_{ij}$  are different matters. A value of  $L_{ij}$  might be estimated by hypothesizing the maximum reasonable amount of damage which might be inflicted by some large number of air strikes, and then using Holliday's method on the "revised" road segment, attempting to account for short-time emergency measures such as shuttling, hand-carrying, etc., which might be employed by the interdicted force. Also, implicit in the estimate of  $L_{ij}$  is the degree to which damage to the road segment can be restored in one day; it might be possible to estimate road capacity immediately after a strike, to estimate the capacity which might be achieved by one day's repair by the interdicted force, and to take the average of these two figures as  $L_{ij}$ .

Estimating values for  $C_{ij}$  is as difficult as estimating values for  $L_{ij}$ . Information on the distributions of  $L_{ij}$  and  $C_{ij}$  could be obtained through tests similar to the RAND Corporation field tests in Thailand in 1962<sup>(7)</sup> by using aircraft to launch strikes against the test roads and determining capacity after each successive strike. Such strikes,

in addition to indicating the maximum extent to which capacity could be reduced and providing information on the distributions of  $L_{ij}$  and  $C_{ij}$ , would also provide an insight into the general behavior of arc capacity with respect to sorties-- whether capacity is a linear function of sorties, or whatever.

Intuition suggests that a plot of arc capacity versus sorties might not be linear (as was assumed in the model), but rather a curvilinear function similar to the solid curve of Figure 4.

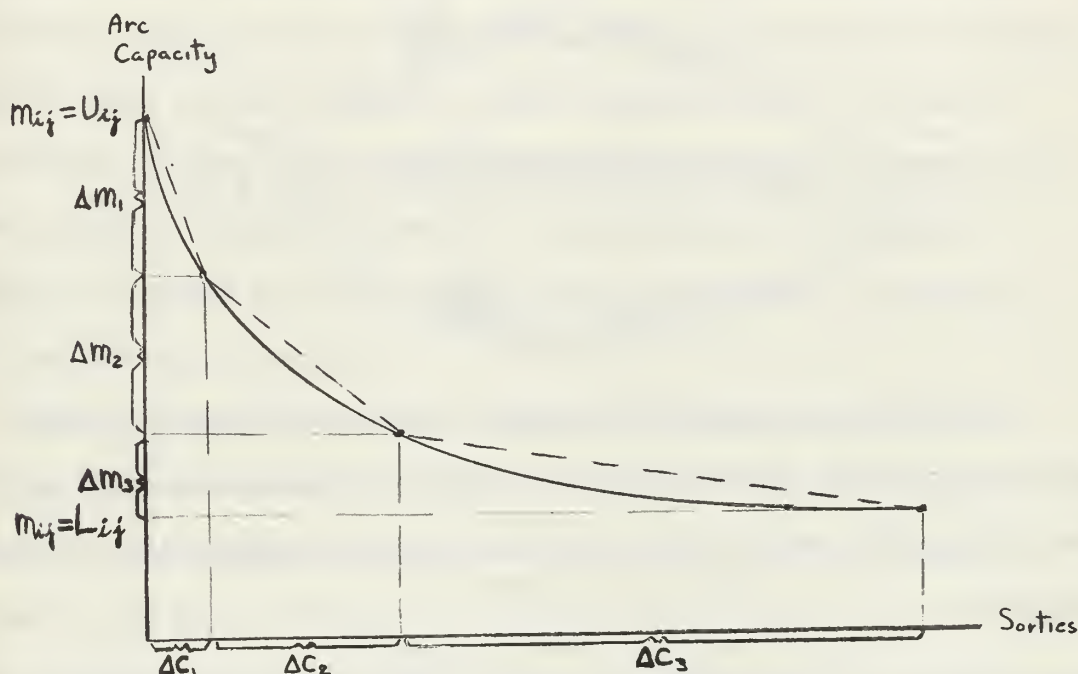
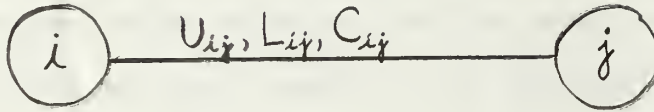


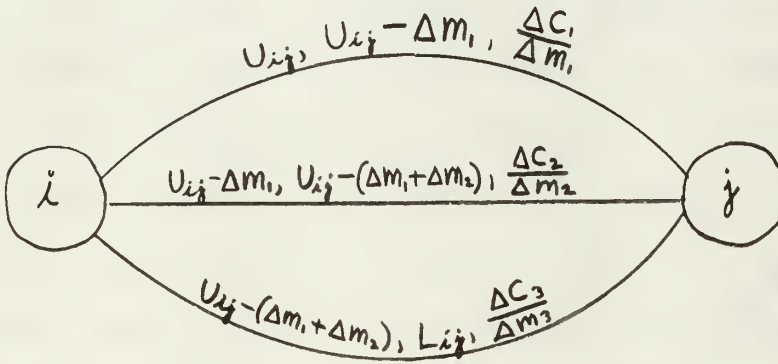
Figure 4. Arc Capacity versus Number of Sorties

However, such non-linear functions can be handled without too much difficulty. The actual curve can be approximated as closely as is desired by a series of piece-wise linear segments (dashed line).

If linearity were assumed, the arc  $(i,j)$  would be represented as



If we wish to incorporate the piece-wise linear approximation to the actual curve, it is only necessary to construct a parallel set of arcs, each corresponding to a straight portion of the dashed line of Figure 4:



By this approximation method, any curve can be incorporated into the model with no change in the procedure of the algorithm, and little increase in the complexity of the solution.

Applications: The results of this paper would seem to be immediately applicable for strike planning purposes in Southeast Asia. In the absence of data from controlled experiments, rough estimates of  $L_{ij}$  and  $C_{ij}$  could be obtainable from experience gained in the Vietnamese war.

The nature of the road and rail complex which comprises the "Ho Chi Minh Trail" and the importation routes into North Viet Nam is probably of a sufficiently simple structure



that strike planning problems could be solved by hand using the SIRL algorithm. However, if this is not the case, the algorithm is readily amenable to being programmed for computer calculation.

Recommendations for Further Study: Two serious drawbacks of the procedure discussed in this paper are the failure to incorporate repair time into the model, and the lack of consideration of aircraft vulnerability. The former discrepancy could most easily be remedied by expanding the time unit considered from one day to, say, one month-- or to whatever length of time is required to encompass the longest repair time of all targets under consideration. Such a procedure should provide a strike profile for the extended time period (e.g., one month), and additional criteria would be needed for determining the day-to-day allocation of the month's sorties.

Similarly, it would be highly desirable to devise a method to account for aircraft vulnerability at various targets. One approach might be to work with estimates of expected number of aircraft lost at each target, and assign some negative utility to the loss of an aircraft. Such a procedure, of course, would require a decision on the commensurability of aircraft losses and network capacity reduction; i.e., how many aircraft is one willing to sacrifice in order to reduce network capacity by a particular amount?

Finally, as discussed previously, additional work is required in the area of determination of  $L_{ij}$  and  $C_{ij}$  values.

## VI. SUMMARY

A method has been devised for determining the optimum allocation of air attack sorties for interdiction of a transportation network, in the instance where the flow of supplies is network-limited. The method is dependent upon estimates of upper and lower bounds on the capacities of the various segments of the network, and upon the estimates of the number of sorties required to effect reduction in capacity from one value to some lower value.

The procedure determines the minimum achievable route length through the topological dual of the network, which is tantamount to the minimum achievable minimum cut set of the primal network, and hence the minimum achievable network capacity. An algorithm is used for determining this route; it functions by considering the shortest route through the dual network and the number of sorties required to achieve it. If that number is greater than the number available, the route length is increased, and the number of sorties concurrently reduced, in the most efficient manner, until the number required equals the number available. This process is repeated for the second shortest route, and the third shortest, etc., until a route is found whose absolute minimum value is in fact achievable, or until a route is found whose absolute minimum value is greater than the minimum achievable length of some previously considered route. The algorithm terminates at this point.

The inputs required for the computational procedure are the total number of sorties available, the maximum

capacity of each arc of the network, the lowest value to which that capacity can be reduced, and the number of sorties required to bring about this reduction. The outputs generated are the set of arcs which should be interdicted, the number of sorties which should be flown against these arcs, and the flow capacity of the network after the sorties have been flown.



## BIBLIOGRAPHY

1. Bellman, R. E., and S. E. Dreyfus, Applied Dynamic Programming, (Princeton: Princeton University Press, 1962).
2. Cagle, M. W., and F. A. Manson, The Sea War in Korea, (Annapolis, Maryland: United States Naval Institute, 1957).
3. Durbin, E. P., An Interdiction Model of Highway Transportation, (The RAND Corporation, RM-4945-PR, May 1966).
4. Field, J. A., History of United States Naval Operations in Korea, (Washington: United States Government Printing Office, 1962).
5. Ford, L. R. Jr., and D. R. Fulkerson, Flows in Networks, (The RAND Corporation, R-375-PR, August 1962).
6. Futrell, R. F., The United States Air Force in Korea 1950-1953, (New York: Duell, Sloan, and Pearce, 1961).
7. Holliday, L. P., A Method for Estimating Road Capacity and Truck Requirements, (The RAND Corporation, RM-3331-ARPA, November 1963).
8. Wollmer, R. D., Some Methods for Determining the Most Vital Link in a Railway Network, (The RAND Corporation, RM-3321-ISA, April 1963).
9. Wollmer, R. D., "Removing Arcs from a Network", Operations Research, Volume 12, number 6, November-December 1964).

# INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2. Library Naval Postgraduate School Monterey, California 93940	2
3. Chief of Naval Operations (OP 96) Navy Department Washington, D. C.	1
4. Asst. Prof. Alan W. McMasters Department of Operations Analysis Naval Postgraduate School Monterey, California 93940	3
5. Lt. Thomas M. Mustin USN c/o 850 B. Avenue Coronado, California	1
6. Asst. Prof. Glenn F. Lindsay Department of Operations Analysis Naval Postgraduate School Monterey, California 93940	1



DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP	
3. REPORT TITLE Optimal Allocation of Air Strikes for Interdiction of a Transportation Network			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Thesis			
5. AUTHOR(S) (Last name, first name, initial) MUSTIN, Thomas Morton, Lieutenant, USN			
6. REPORT DATE June 1967		7a. TOTAL NO. OF PAGES 42	7b. NO. OF REFS 9
8a. CONTRACT OR GRANT NO.		8a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO.			
c.		8b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.			
10. AVAILABILITY/LIMITATION NOTICES <del>ALL INFORMATION CONTAINED HEREIN IS UNCLASSIFIED EXCEPT WHERE SHOWN OTHERWISE</del> <del>UNCLASSIFIED FOR FOREIGN NATIONALS MAY BE MADE ONLY WITH PRIOR</del> <del>APPROVAL OF THE NAVAL POSTGRADUATE SCHOOL</del>			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California	
13. ABSTRACT <p>Under certain conditions, the re-supply capability of a combatant force may be limited by the characteristics of the transportation network over which supplies must flow. Air strikes by an opposing force may be used to reduce the capacity of that network; the effects of such strikes vary for differing missions and targets. With only a limited number of sorties available, the strike planner must decide which targets to hit, and with how many sorties. A computational procedure is developed for determining the optimum strike plan for minimizing network flow capacity.</p>			

### KEY WORDS

## Interdiction

Transportation

LINK C

WT









thesM9865

Optimal allocation of air strikes in



2 2768 000 99371 1

DUDLEY KNOX LIBRARY